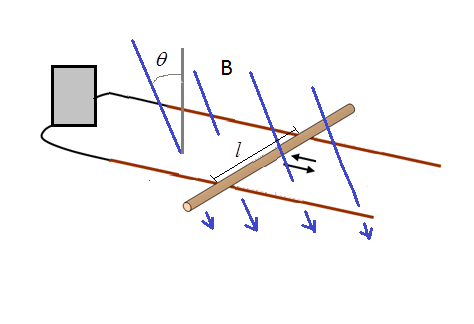
Motor and Generator Problems

**Question 9**. Suppose you’ve connected your cell phone to a generator which consists of parallel conducting rails and 10cm long sliding wire. You bought a 1.2T magnet on Ebay and have immersed the sliding wire in its magnetic field (pointed at a 0° w/r to the vertical). You shake the sliding wire back and forth (perhaps by hooking it up to the wheels of a stationary bike) so that its position as a function of time is given by x(t) = 5sin(2πft) (*cm*). What frequency, f, would be sufficient to generate a maximum potential difference of 6V, enough to charge, say, your cell phone? If the resistance of your cell phone is 20Ω, what maximum force would the magnetic field exert on the sliding wire?



So we have:

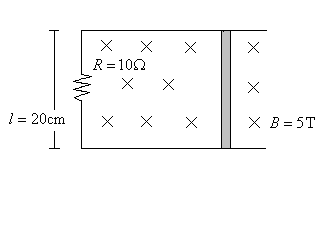


Force exerted is:



**Problem**

How fast and in which direction must you move the bar if you want the current to circulate CCW with magnitude 1A?



**Solution**

Magnitude of current is given by,

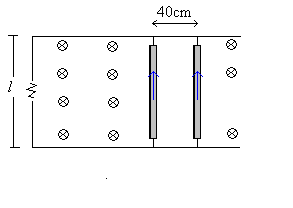


Must pull bar to right; in that case the force on the charges in the rod will be upwards, since **v**×**B** will be going up.



**Problem**

Suppose you need a battery, but don’t have one and so you decide to improvise. You take a ℓ = 10cm long wire and wiggle it back and forth through an amplitude of range of 40cm with a period of 0.5s in Earth’s magnetic field of ~ 50μT. What potential difference do you generate as a function of time? What is the maximum potential difference generated?



**Solution**

Well, the position of the bar will be



A is 20cm, and ω = 2πf = 2π/T = 2π/0.5 = 12.6 Hz. So,



and so,

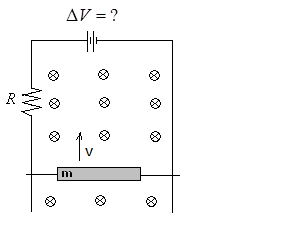


and thus,



**Problem**

Consider the following electric motor, designed to raise the bar with mass m = 5kg at a speed of 2m/s. To accomplish this, what potential difference need the battery supply, supposing that B = 10T, and R = 0.20Ω, and the length of the wire is ℓ = 30cm?



**Solution**

The battery will set up a current in the wire, labelled I. Thus the magnetic field will exert a force on the wire given by: FB = ℓIB, which will be opposed by gravity, Fg = mg. Since the bar moves at a constant velocity, we must have,



Now the current is given by I = ΔV/R, where ΔV comes from two parts. One is the contribution of the battery, V, and the other is from the back emf resulting from the motion of the bar through the magnetic field. This is Vback = Bℓv. This emf will oppose that of the battery so we have, so we have,



equating these two expressions we get,



**Example**

Suppose we knew V, and wanted B. What would we get:



Why two solutions. Consider going backwards. If I have either B, what will V need to be? It appears there are indeed two legitimate solutions. As m → 0, we’d have: B → V/ℓv, 0. Second is true if holding bar. Second is true if

Suppose we send v → 0. Then in case where we want to simply hold the bar, we should get V =

IR, ℓIB = mg → B = mg/ℓI = mg/ℓ(V/R) = mgR/ℓV, which is second solution. The first would give you V/ℓv → ∞.

**Example**

Let’s do the bar example as a function of time. We have:



Substituting B we have:



Let’s define:



Can write:



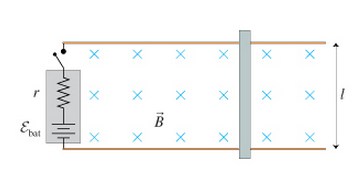
And so,



Formula predicts that as t → ∞. Must have v → v\*. So v\* is limiting velocity. If start at v0 = v\*, then, formula breaks down.

**Problem**

A projectile (rod) is to be launced by a battery. The mass of the projectiel is m = 20kg., battery’s emf is εbat = 100kV and its internal resistance is r = 0.4Ω. What will be the terminal velocity of the bar? How long will it take to accelerate the bar up to 95% terminal speed (i.e. 3 time constants?)



Doing Kirchoff’s law around the loop we have:



Now at the terminal velocity, the force on the net force on the bar will be zero, which means

F­B = 0 → ℓIB = 0 → I = 0. So then filling this into the equation we get:

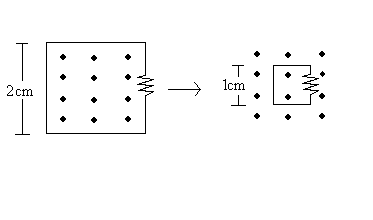


We will need to know the acceleration of the rod to know the distance. We have:



So 3τ = 3mr/ℓ2B = 3(20)(0.4)/(0.25)2(4) = 96s.

13. Suppose you have a metallic square of side lengths 2cm and resistance 4Ω, in a magnetic field of 10T pointing out of the page. If you shrink the square so that it has side lengths of 1cm, and you do this is 5s, what will be the magnitude and direction of the induced current?



We can use,



And then, the current is:



the direction of the EMF is CCW so the current is too.

**Problem**

Suppose you stack 10 wires squares of length 20cm on top of eachother and connect it to your bike. If you pedal twice every second, what maximum emf can you generate from the wire?

**Solution**

Well, the maximum potential difference you can generate is:

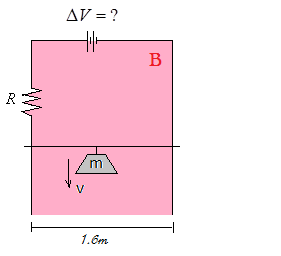


N = 10. B is the ambient magnetic field is Earth’s, which is around 50μT. A is 0.122, and ω is 2rev/s = 2(2π) rad/s. So putting these together comes to:



Not too much for sure.

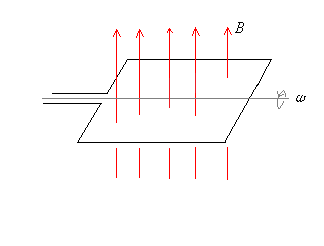
**Question 10**. You want to *lower* a 50kg weight at a constant velocity of v = 2.5m/s. The resistance of your circuit is R = 20mΩ, and the magnetic field strength is B = 0.50T. What potential difference is required, and what direction should B be pointed in (left/right, up/down, into/out of the page)? Pay careful attention to signs when doing KVL here.



Need the force to be up, to counter gravity, and so if current is to the right across the sliding wire, then need B to be into page again. From N2L we have: FB = Fg → ℓIB = mg → I = mg/ℓB. Then from KVL we have +ΔV – IR + Bℓv = 0 (note ΔVeff = Bℓv points to the *right* here b/c v is *down*) → ΔV – IR – Bℓv = mgR/ℓB – Bℓv = (50)(9.8)(0.020)/(1.6)(0.50) – (0.50)(1.6)(2.5) = 10.25V.

**Problem**

You are charging your cellphone using a home-made contraption. You attach a stack of wire loops to your bicycle wheel. There are 50 wire loops in the stack. Each one is a square with side length ℓ = 20cm, and are placed in a magnetic field of strength B = 0.6T. If you pedal at rate of 100 revolutions/minute, what is the maximum effective potential difference will you generate in the loops overall?



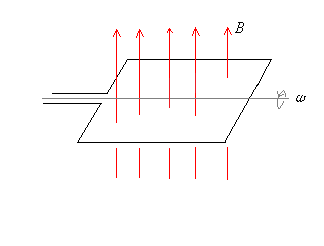
**Solution**

The effective potential is given by Veff. = NBAωsin(ωt). Therefore the amplitude is

Vmax = NBAω = (50)(0.6)(0.22)(100∙2π/60) = 12.6 V.

**Problem**

You are charging your cellphone using a home-made contraption. You attach a stack of wire loops to your bicycle wheel. There are 50 wire loops in the stack. Each one is a square with side length ℓ = 20cm, and are placed in a magnetic field of strength B = 0.6T. If you pedal at rate of 100 revolutions/minute, what maximum effective potential difference will you generate in the loops overall?



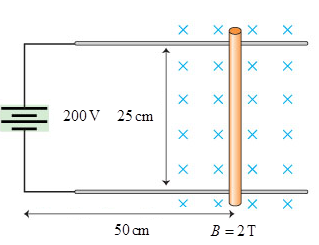
Suppose you connect these wire loops to your cell phone which has a resistance of 1500Ω. What maximum current will be running through the wires in this case? What is the maximum counter-torque the magnetic field will exert on these wires? FYI, this would be the torque you’d have to exert via the pedals to turn the wires in the first place, and it’s not very much.

**Solution**

The max current will be imax = Vmax/R = 12.6/1500 = 8.4 mA. The counter torque is:

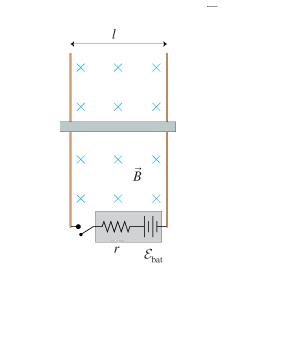


**Question 2**. A device called a *railgun* uses the magnetic force on currents to launch projectiles at very high speeds. An idealized model of a railgun is illustrated in the figure. A power supply is connected to two conducting rails. A segment of copper wire, in a region of uniform magnetic field, slides freely on the rails. The wire has a 0.28mΩ resistance and a mass of 75kg. (a) What will be the force on the segment at t = 0? (b) What will be its terminal velocity?



Force on wire will be F = ℓIB = ℓ(V/R)B = (0.25)(200/0.28×10-3)2 = 3.57×105N. Its terminal velocity will occur when F = 0 → I = 0 → ΔV = 0 → 200 – Bℓv = 0 → v = 200/Bℓ = 200(2×0.25) = 400m/s.

**Question 10**. A 30V battery with internal resistance r = 25mΩ is connected to a slide rail supporting a metallic rod of length ℓ = 20cm and mass m = 18kg. If B = 0.75T, with what constant speed will the magnetic field push the rod up the rails?



If the rod is being pushed upwards at a constant speed, then the magnetic force on the rod will equal the gravitational force. So:

